**Fano 2-102-27-a.** $X = \mathscr{Z}(\mathrm{Gr}(3,8),\, \bigwedge^2 \mathcal{Q}^\vee(1) \oplus \mathcal{O}(1))$.

*Invariants.* $h^0(-K_X) = 27, \quad (-K_X)^4 = 102, \quad h^{1,1} = 2, \quad h^{3,1} = 1, \quad h^{2,2} = 22, \quad -\chi(T_X) = 19.$

*Description.* **This is the Küchle fourfold of type (c7)**, introduced in [Kü95] and studied by Kuznetsov in [Kuz15]. Let $S = \nu_2(\mathbb{P}^2) \subset \mathbb{P}^5$ be a Veronese surface, and let $Y \subset \mathbb{P}^5$ be a general smooth cubic fourfold containing $S$. Write $H$ for the pullback of $\mathcal{O}_Y(1)$ and $E$ for the exceptional divisor of the blow-up $f\colon X \to Y$.

(1) $f\colon X \simeq \mathrm{Bl}_S Y \to Y$ is a $K$-negative elementary divisorial contraction [Kuz15, Corollary 4.11].
(2) The quadratic Cremona transformation of $\mathbb{P}^5$ centered at $S$ induces a second $K$-negative elementary divisorial contraction $f'\colon X \simeq \mathrm{Bl}_{S'} Y' \to Y'$, where $S' \subset Y' \subset \mathbb{P}^5$ is another Veronese surface contained in a smooth cubic fourfold [Kuz15, Lemma 4.1 and Corollary 4.11].

- $H' = f'^*\mathcal{O}_{Y'}(1) = 2H - E$, $-K_X = H + H'$, and $\mathrm{Nef}(X) = \langle H, H' \rangle$ is simplicial.

*Semiorthogonal decompositions.* By [Kuz15, Corollary 4.12],

$$\mathrm{D}^{\mathrm{b}}(X) = \langle \mathcal{O}_X, \mathcal{O}_X(H), \mathcal{O}_X(2H),\, i_*\mathcal{O}_E,\, i_*\mathcal{O}_E(h),\, i_*\mathcal{O}_E(2h),\, \mathcal{A}_X \rangle,$$

where $i\colon E \hookrightarrow X$, $h$ is the pullback of the positive generator of $\mathrm{Pic}(S)$, and $\mathcal{A}_X$ is equivalent to the Kuznetsov K3 category of $Y$.

◇

## References


[Kü95] O. Küchle, *On Fano 4-folds of index 1 and homogeneous vector bundles over Grassmannians*, Math. Z. **218** (1995), no. 4, 563–575.

[Kuz15] A. Kuznetsov, *On Küchle varieties with Picard number greater than* 1, Izv. Math. **79** (2015), no. 4, 698–709.